

Constraints on R^n gravity from precession of orbits of S2-like stars: a case of a bulk distribution of mass

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Abstract

Here we investigate possible applications of observed stellar orbits around Galactic Center for constraining the R^n gravity at Galactic scales. For that purpose, we simulated orbits of S2-like stars around the massive black hole at Galactic Center, and study the constraints on the R^n gravity which could be obtained by the present and next generations of large telescopes. Our results show that R^n gravity affects the simulated orbits in the qualitatively similar way as a bulk distribution of matter (including a stellar cluster and dark matter distributions) in Newton's gravity. In the cases where the density of extended mass is higher, the maximum allowed value of parameter β in R^n gravity is noticeably smaller, due to the fact that the both extended mass and R^n gravity cause the retrograde orbital precession.

Keywords: Black hole physics; Galactic Center; Astrometry; Alternative theories of gravity

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1. Introduction

Dark matter (DM) (Zwicky, 1933) and Dark Energy (Turner, 1999) problems are fundamental and difficult for the conventional General Relativity approach for gravity, (Zakharov et al., 2009) see also the monograph by Weinberg (2008) for a more comprehensive review.

There is an opinion that an introduction of alternative theories of gravity (including so-called $f(R)$ theories (Capozziello and Fang, 2002; Capozziello et al., 2003, 2006; Carroll et al., 2004; Capozziello et al., 2007; Capozziello and Faraoni, 2012; Mazumdar and Nadathur, 2012)) could after all give explanation of observational data without DM and DE problems. However, a proposed gravity theory has to explain not only cosmological problems but many other observational data because sometimes these theories do not have Newtonian limit for a weak gravitational field case, so parameters of these theories have to be very close to values which correspond to general relativity (Zakharov et al., 2006). Earlier, constraints on $f(R) = R^n$ have been obtained from an analysis of trajectories of bright stars near the Galactic Center (Borka et al., 2012, 2013), assuming a potential of bulk distribution of matter is negligible in comparison with a potential of a point like mass. In this paper we consider modifications of results due to a potential of a bulk distribution of matter assuming that this potential is small in comparison with point like mass one. For the standard GR approach these calculations have been done (Zakharov et al., 2007; Nucita et al., 2007).

We would like to mention that not only trajectories of bright stars but also probes such as LAGEOS and LARES (Ciufolini and Pavlis, 2004; Ciufolini, 2007) could provide important test for alternative theories of gravity, see for instance constraints on the Chern–Simons gravity (Smith et al., 2008).

2. Method

We simulated orbits of S2 star around Galactic Center in the R^n gravity potential, assuming a bulk distribution of mass in the central regions of our Galaxy. The R^n gravity potential is given by Capozziello et al. (2006, 2007):

$$\Phi(r) = -\frac{GM_{BH}}{2r} \left[1 + \left(\frac{r}{r_c} \right)^\beta \right], \quad (1)$$

where r_c is an arbitrary parameter, depending on the typical scale of the considered system and β is a universal constant depending on n (Capozziello

et al., 2006; Zakharov et al., 2006)

$$\beta = \frac{12n^2 - 7n - 1 - \sqrt{36n^4 + 12n^3 - 83n^2 + 50n + 1}}{6n^2 - 4n + 2}. \quad (2)$$

We use two component model for a potential in the central region of our Galaxy which is constituted by the central black hole of mass $M_{BH} = 4.3 \times 10^6 M_\odot$ (Gillessen et al., 2009) and an extended distribution of matter with total mass $M_{ext}(r)$ (including a stellar cluster and dark matter) contained within some radius r . For the density distribution of extended matter we adopted the following broken power law proposed by Genzel et al. (2003):

$$\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\alpha}, \quad \alpha = \begin{cases} 2.0 \pm 0.1, & r \geq r_0 \\ 1.4 \pm 0.1, & r < r_0 \end{cases} \quad (3)$$

where $\rho_0 = 1.2 \times 10^6 M_\odot \cdot \text{pc}^{-3}$ and $r_0 = 10''$. This leads to the following expression for the extended mass distribution:

$$M_{ext}(r) = \frac{4\pi\rho_0 r_0^\alpha}{3-\alpha} r^{3-\alpha}. \quad (4)$$

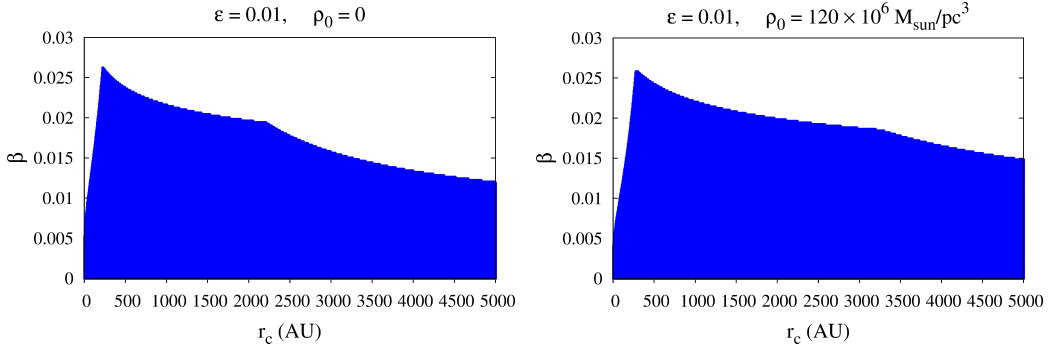


Figure 1: Parameter space for R^n gravity with different contributions of extended mass under the constraint that, during one orbital period, S2-star orbits in R^n gravity differ less than 10 mas ($\varepsilon = 0''.01$) from its Keplerian orbit. The assumed values for mass density constant ρ_0 from Eq. (3) are denoted in the title of each panel.

The corresponding potential for extended distribution of matter is then:

$$\Phi_{ext}(r) = -G \int_r^{r_\infty} \frac{M_{ext}(r')}{r'^2} dr' = \frac{-4\pi\rho_0 R_0^\alpha G}{(3-\alpha)(2-\alpha)} (r_\infty^{2-\alpha} - r^{2-\alpha}), \quad (5)$$

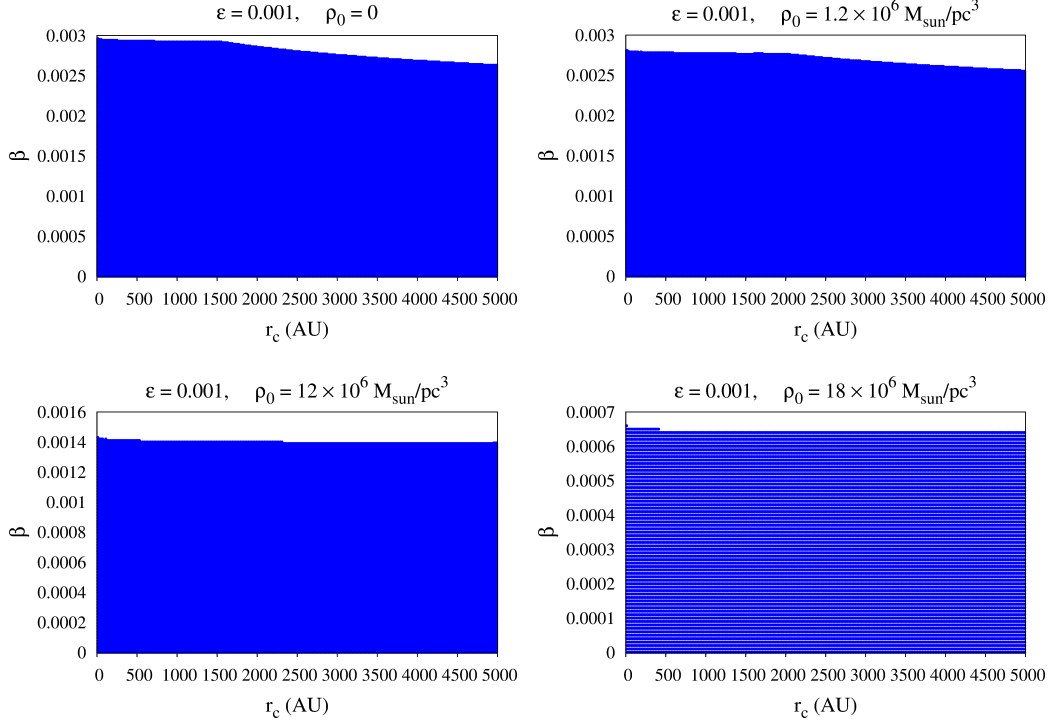


Figure 2: The same as in Fig. 1, but for 10 times higher astrometric precision of 1 mas ($\epsilon = 0''.001$).

where r_∞ is the outer radius for extended distribution of matter which is enclosed within the orbit of S2 star. The total gravitational potential for the two component model can be evaluated as a sum of R^n potential for central object with mass M_{BH} and potential for extended matter with mass $M_{ext}(r)$:

$$\Phi_{total}(r) = \Phi(r) + \Phi_{ext}(r). \quad (6)$$

Thus, the simulated orbits of S2 star could be obtained by numerical integration of the following differential equations of motion in the total gravitational potential:

$$\dot{\mathbf{r}} = \mathbf{v}, \quad \ddot{\mathbf{r}} = -\nabla (\Phi_{total}(\mathbf{r})). \quad (7)$$

3. Results

In Figs. 1–3 we present the parameter space for R^n gravity with different contributions of extended mass for which the discrepancies between the orbits

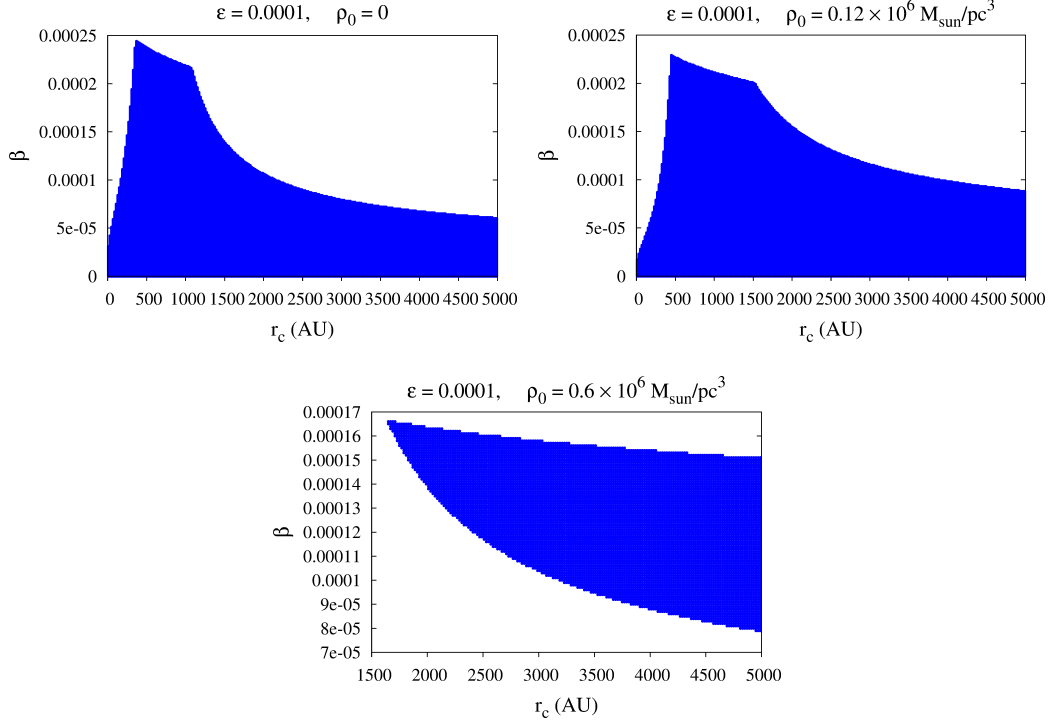


Figure 3: The same as in Fig. 1, but for 100 times higher astrometric precision of 0.1 mas ($\epsilon = 0''.0001$).

of S2-star in R^n gravity and its Keplerian orbit during one orbital period are less than an assumed astrometric precision. As it can be seen from Fig. 1, it is very difficult to detect the contribution of extended mass with the astrometric precision of 10 mas, which was the actual limit during the first part of the observational period of S2 star, some 10–15 years ago. The blue area of parameter space changing very little with variations of ρ_0 . However, with the current astrometric limit reaching less than 1 mas, this contribution significantly constrains the maximum allowed value β (see Fig. 2). In the cases where the density of extended mass is higher, the maximum allowed value of β is noticeably smaller, due to the fact that the both extended mass and R^n gravity have the similar effect on S2 star orbit, i.e. they both cause the retrograde orbital precession. However, the astrometric limit is constantly improving and in the future it will be possible to measure the stellar positions with much better accuracy of $\sim 10 \mu\text{as}$ (Gillessen et al., 2010). The parameter space for currently unreachable accuracy of 0.1 mas

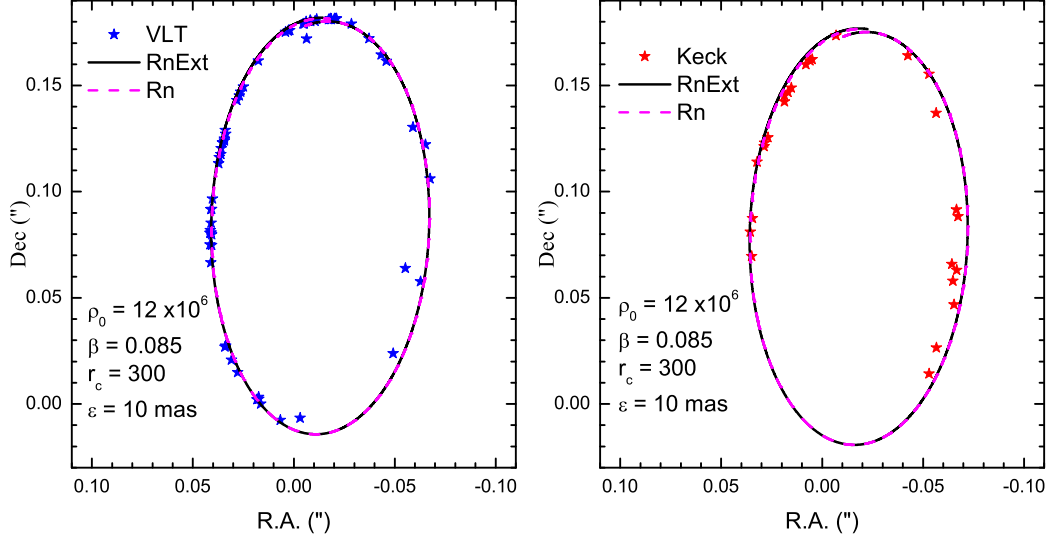


Figure 4: Comparison between the simulated orbits of S2 star in the R^n gravity potential with (solid line) and without (dashed line) contribution of extended mass, and NTT/VLT (left) and Keck (right) astrometric observations. The parameters of R^n gravity are obtained by fitting the simulated orbits in total potential to the observations, assuming astrometric accuracy of 10 mas and the density constant $\rho_0 = 12 \times 10^6 M_\odot \cdot \text{pc}^{-3}$.

is presented in Fig. 3, from which one can see that even a small amount of extended mass would practically exclude the R^n term from the total gravity potential. Even more, we found that in such a case expression (3) would result with overestimated amount of extended mass at Galactic center, and therefore, we assumed 2 and 10 times smaller densities ρ_0 . Besides, from Figs. 1–3 it is obvious that both astrometric precision and amount of extended mass have significant influence on the value of r_c for which maximum β is expected. For example, in the top right panel of Fig. 1 the maximum $\beta \approx 0.027$ is expected for $r_c \approx 250$ AU, while in the top right panel of Fig. 3 the maximum $\beta \approx 0.00023$ is expected for $r_c \approx 450$ AU.

Several comparisons of the simulated S2 star orbits in the R^n gravity potential with and without contribution of extended mass with NTT/VLT and Keck astrometric observations are given in Fig. 4, assuming the astrometric accuracy of 10 mas. In this case, influence of extended mass for $\rho_0 = 1.2 \times 10^6 M_\odot \cdot \text{pc}^{-3}$ (Genzel et al., 2003) is almost negligible, and hence it cannot explain the observed precession of S2 star orbit. Therefore, in order to explain the observed precession, either a higher value of β in R^n gravity

potential (see e.g. [Borka et al., 2012](#)), or much higher density of extended mass are necessary (as it is the case in Fig. 4).

4. Conclusions

In this paper we analyze stellar orbits around Galactic Center in the R^n gravity with extended mass distribution. Our results show that R^n gravity with extended mass distribution could significantly affect the simulated orbits. Both R^n gravity and extended mass distribution give retrograde direction of the precession of the S2 orbit. In the cases if the density of extended mass is higher, the maximum value of β which is consistent with observations in R^n gravity is noticeably smaller. We confirmed that the R^n gravity parameter β must be very close to those corresponding to the Newtonian limit of the theory. When parameter β is vanishing, we recover the value of the Keplerian orbit for S2 star. When we take into account extended mass distribution, parameter β is less than in case without extended mass distribution and faster approaching to zero. Even more, one can see that relatively small amount of extended mass would practically exclude the R^n term from the total gravity potential.

We can conclude that both effects, additional term in R^n gravity and extended mass distribution, produce a retrograde shift, that results in rosette shaped orbits. Also, we can conclude that both astrometric precision and extended mass distribution have significant influence on the value of r_c for which maximum β could be expected.

Although both observational sets (NTT/VLT and Keck) indicate that the orbit of S2 star might not be closed, the current astrometric limit is not sufficient to unambiguously confirm such a claim. However, the astrometric accuracy is constantly improving from around 10 mas during the first part of the observational period, currently reaching less than 1 mas.

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References

Borka D., Jovanović P., Borka Jovanović V., Zakharov A.F., Constraints on R^n gravity from precession of orbits of S2-like stars, *Phys. Rev. D* **85**, 2012,

124004-1–124004-11.

- Borka D., Jovanović P., Borka Jovanović V., Zakharov A.F., Constraining the range of Yukawa gravity interaction from S2 star orbits, *J. Cosm. Astropart. Phys.* **11**, 2013, 050-1–050-15.
- Capozziello, S., Cardone, V.F. & Troisi, A., Gravitational lensing in fourth order gravity, *Phys. Rev. D* **73**, 2006, 104019-1–104019-10.
- Capozziello, S., Cardone, V.F., & Troisi, A., Low surface brightness galaxy rotation curves in the low energy limit of R^n gravity: no need for dark matter? *Mon. Not. R. Astron. Soc.* **375**, 2007, 1423–1440.
- Capozziello S., Fang L.Z., Curvature Quintessence, *Int. J. Mod. Phys. D* **11**, 2002, 483–491.
- Capozziello S., Cardone V.S., Carloni S. Troisi A., Curvature Quintessence Matched with Observational Data, *Int. J. Mod. Phys. D* **12**, 2003, 1969–1982.
- Capozziello S. and Faraoni V., *Beyond Einstein Gravity: A Survey Of Gravitational Theories For Cosmology And Astrophysics*, 2011, Springer, New York.
- Carroll S.M., Duvvuri V., Trodden M., Turner M.S., Is cosmic speed-up due to new gravitational physics? *Phys. Rev. D* **70**, 2004, 043528-1–043528-5.
- Ciufolini I., Dragging of inertial frames, *Nature* **449**, 2007, 4147.
- Ciufolini I. and Pavlis E., A confirmation of the general relativistic prediction of the Lense-Thirring effect, *Nature* **431**, 2004, 958-960.
- Genzel R., Ott T., Eisenhauer F., et al., The stellar cusp around the supermassive black hole in the Galactic Center, *Astrophys. J.* **594**, 2003, 812–832.
- Gillessen S., Eisenhauer F., Fritz T.K., Bartko H., Dodds-Eden K., Pfuhl O., Ott T. & Genzel R., The orbit of the star S2 around Sgr A* from Very Large Telescope and Keck data, *Astrophys. J.* **707**, 2009, L114–L117.

- Gillessen S., Eisenhauer F., Perrin G., et al., GRAVITY: a four-telescope beam combiner instrument for the VLTI, *Proc. of SPIE* 2010, 7734-1–7734-20.
- Mazumdar A. and Nadathur S., Curvaton Scenario within the Minimal Supersymmetric Standard Model and Predictions for Non-Gaussianity, *Phys. Rev. Lett.* **108**, 2012, 111302-1–111302-4.
- Moffat J. M., Gravitational theory, galaxy rotation curves and cosmology without dark matter, *J. Cosm. Astropart. Phys.* **05**, 2005, 012-1–012-20.
- Nucita A.A. et al., Sgr A*: A Laboratory to Measure the Central Black Hole and Stellar Cluster Parameters, *Publ. Astron. Soc. Pac.* **119**, 2007, 349–359.
- Smith T.L., Erickcek A.L., Caldwell R.R., and Kamionkowski M., Effects of Chern-Simons gravity on bodies orbiting the Earth, *Phys. Rev. D* **77**, 2008, 024015-1–024015-9.
- Turner M. S., Dark Matter and Dark Energy in the Universe, In *The Third Stromlo Symposium: The Galactic Halo*, eds. Gibson, B.K., Axelrod, T.S. & Putman, M.E., ASP Conference Series Vol. 165, 1999, pp. 431–452.
- Weinberg S., *Cosmology*, 2008, Oxford University Press.
- Zakharov A. F., Nucita A. A., De Paolis F., Ingrosso G., Solar system constraints on R^n gravity, *Phys. Rev. D* **74**, 2006, 107101-1–107101-4.
- Zakharov A. F., Nucita A. A., De Paolis F., Ingrosso G. Apoastron shift constraints on dark matter distribution at the Galactic Center, *Phys. Rev. D* **76**, 2007, 062001-1–062001-6.
- Zakharov A.F., Capozziello S., De Paolis F., Ingrosso G., Nucita A.A., The Role of Dark Matter and Dark Energy in Cosmological Models: Theoretical Overview, *Space Sci. Rev.* **48**, 2009, 301–313.
- Zwicky F., Die Rotverschiebung von extragalaktischen Nebeln, *Helv. Phys. Acta* **6**, 1933, 110–127.

$\varepsilon = 0.01, \quad \rho_0 = 0.12 \times 10^6 M_{\text{sun}}/\text{pc}^3$

